Income-Share Agreements

A Mechanism Design Approach

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May 10, 2020

There is a growing interest in the use of income-share agreements to finance higher education. This paper aims to offer the first game-theoretic model of how such ISAs could affect the incentives of students to acquire human capital, and the incentives of universities to supply education. I find that ISAs highlight a tension between different policy goals: relative to tuition pricing, ISAs increase the availability of education, but reduce student productivity. In a duopoly environment, where both universities can choose a pricing model, the dominant strategy equilibrium is for both universities to choose tuition pricing. Furthermore, this equilibrium is efficient: tuition pricing maximizes both the availability of education and the productivity of education, so it is socially superior to ISA pricing.

Acknowledgements: This paper is my undergraduate economics honors thesis. I am grateful to my advisor Rakesh Vohra for tirelessly guiding this paper, and to Jere Behrman and the ECON 300 seminar participants for helpful comments on early versions of this paper. Special thanks are due to Clayton Featherstone, Judd Kessler, Annie Liang, Ben Golub and Scott Kominers whose guidance has been instrumental for my development as a researcher.

Contents

1. Introduction 3

2. Literature Review 7

3. Tuition Pricing 8

3.1. Interior-Optimal Mechanisms . . . 10

3.2. Shut-Out Mechanisms . . . 13

4. Income-Share Agreements 15

4.1. ISA Optimum . . . 16

4.2. Optimal Effort Under ISAs . . . 17

4.3. Social Welfare Under ISAs . . . 18

5. Duopoly 19

5.1. Tuition Duopoly . . . 20

5.2. Tuition-ISA Duopoly . . . 21

5.3. ISA Duopoly . . . 22

5.4. Duopoly Subgame-Perfect Equilibrium . . . 23

6. Conclusion 25

A. Type Report Independence 29

A.1. Monopoly . . . 29

A.2. Duopoly . . . 30

B. Monopoly Interior Optimum 30

C. Duopoly Proofs 34

1. Introduction

In 1955, Milton Friedman proposed "equity investment contracts" in individuals seeking higher education, in which students finance their education by giving lenders an "equity stake" in their future earnings. (Friedman 1955) Today, that concept is rapidly becoming reality with the growth of income-share agreements: a financing instrument for higher education in which a student promises a fraction of their future earnings instead of a fixed tuition amount. Private universities in Germany offer ISAs, and postgraduate degrees in the UK are often funded by ISAs from StepEx, an accredited private financier taken up by many students. (FT 2019) Coding bootcamps like the Lambda School have also flourished on the ISA model, advertising that they do not take any fees and students do not pay until they earn their first paycheck. ISAs are also growing in popularity in American universities: Purdue University and the University of Utah now offer ISA funding options to their students, and the US government is piloting an experimental program to allow student debt to be repaid under an ISA model. (SLH 2019)

ISAs have emerged from the growing social dissatisfaction with the tuition fee model that is prevalent in universities throughout the world. In the US, median family income has grown 22% between 1970 and 2010: in that same timeframe, public tuition has grown by 200%, and private tuition has grown by 150%. (Kirshstein 2012) In this context, proponents of ISAs argue that they increase the accessibility of education relative to tuition fees, and thus policymakers should consider adopting ISAs at larger scale. (Palacios, DeSorrento, and Kelly 2014)

In light of the growing prominence of ISAs and their potential importance, this paper seeks to compare tuition fees to ISAs as a means to finance higher education. Specifically, we focus on two questions: first, how do ISAs affect the incentives of universities to supply education? Second, how do ISAs affect the incentives of students to invest in their own productivity? These two questions allow for a coherent discussion of how ISAs affect the education system's efficiency, and the welfare of society as a whole.

In seeking to answer these two questions, two challenges prevent us from relying on an empirical approach to ISAs. The first challenge is conceptual: *incentives* cannot be directly observed in data, because they affect both the decisions that students and universities take (which can be observed) as well as the decisions that students and universities choose not to take (which cannot be observed). The second challenge is logistical: even if incentives could be observed in the data, data on ISAs in education is very difficult to access due to the small number of existing

ISAs, as well as the natural confidentiality of individual student data. Thus, in order to answer the question we pose, it is necessary to develop a game-theoretic model of how universities and students behave. This game-theoretic model will allow us to make predictions about outcomes we would expect to observe if the model holds true.

The model has three parts. The first part is the university: we take a university to be a profit-maximizing entity that has a fixed capacity of seats for students. To begin with, we take the university to be a monopolist: this captures the general idea that universities have bargaining power over students, and is relaxed in later sections. The university faces a population of students that exceeds the capacity of the university: thus, the university is supply-constrained and the availability of education depends on whether the university chooses to allocate all its seats or not.

The second part of the model is a representation of students. There are two types of students, representing two kinds of students that are most salient when studying the financing of higher education. The first type, known as **high-type** students, are students with high expected wages on the labor market. Similarly, the second type, known as **low-type** students, are students with lower expected wages on the labor market.[1] Students know whether they are high-type or low-type. Furthermore, this knowledge is private: the university cannot observe it directly, but rather must rely on student self-reports (in the form of college applications) to determine a student's type. Students announce their type to the university, and the university chooses an admissions rate for each announced type.

The third part of the model is the university education itself. Once admitted to the university, students exert some private effort that can increase their productivity, converting them to a high-type student with some probability proportional to the effort they invest. This feature represents the possibility of social mobility through education. Once students leave the university, they receive a labor market wage that reflects their true type. Importantly, neither high-type nor low-type students can access these labor market wages without going to university: all students, regardless of their type, have the same "outside option", their alternative to university. This reflects a credentialist attitude in society, where a university degree is often demanded by employers to signal ability.

[1]This type system is consistent with multiple interpretations of what causes earnings differences between students—it could represent high inherent-ability and low-inherent ability students, or high-privilege and low-privilege students, or any other such interpretation. This model is agnostic as to the cause of students having different labor market wages: it only assumes that such differences exist.

These facts define the base model, but they leave unspecified how the university makes money: in other words, how students are charged for university. We consider two options for this pricing scheme: tuition fees, and income-share agreements. Under tuition pricing, the university chooses a tuition fee for every student (which may depend on their announced type), that students pay if they attend the university. In contrast, under income-share agreements, the university chooses a fraction of future wages to demand of all admitted students. Note that since wages are based on true type rather than announced type, the university's revenue does not depend on the student's announced type. Thus, ISA pricing differs from tuition pricing along two dimensions: it depends on the student's true hidden type rather than their announced type. and the fraction of income demanded does not vary by type, whether announced or true.

From this model, the major results of the paper follow. When the university is a monopolist, students are generally worse off under ISAs, because ISAs enable the university to charge higher rates without risk of students turning away. Furthermore, relative to tuition fees, ISAs decrease the incentives of students to invest in their productivity: under the university's optimal ISA, students do not invest in their productivity at all. This makes the education system inefficient for each student.

On the other hand, tuition pricing incentivizes universities to restrict supply from low-type students, whereas ISA pricing incentivizes the university to allocate all seats. As a result, ISAs increase the supply of education relative to tuition fees, which contributes to educational efficiency. Thus, proponents of ISAs are correct in arguing that ISAs increase the availability of education. However, they ignore the negative effect ISAs have on students themselves. In this way, ISAs highlight a tension between different efficiency criteria in the education system. Highlighting this tension between different policy goals is one of the major contributions of this paper, and is made possible through the game-theoretic analysis that accounts for the clashing incentives of universities and students.

This result implies ISAs are more profitable for universities, but that begs the question of why tuition fees are almost universal. However, we can explain this nuance by relaxing the assumption that the university is a monopolist, and instead moving to a duopoly environment. Duopoly models have long been used in economics to capture the effects of imperfect competition more generally, which is a reasonable representation of the higher education market. In a duopoly environment, we can consider an expanded game where the university's choice of pricing model depends on its competitor's choice of pricing model. Universities play a dynamic, two-stage game: in the first stage, they simultaneously adopt either tuition pricing or ISA pricing. In the second stage, they adopt optimal policies given their pricing model and the pricing model of their competitor. We analyze this duopoly with the concept of a *subgame-perfect equilibrium*, an adaptation of the well-known concept of Nash equilibrium to dynamic games.

We find that in this game, it is a dominant strategy for a university to adopt tuition fees—no matter what model its competitor adopts, a university sees higher profit from adopting a tuition model. After adopting fees, the universities enter a *sorting equilibrium*, in which one university accepts only high-type students, the other university accepts only low-type students, and all students are charged a uniform fee. This uniform fee is set at the highest price that a low-type student can pay. This equilibrium explains why tuition fees are so prevalent in universities around the world. Furthermore, the sorting equilibrium is consistent with observations of real higher education, where tuition fee variance is small across universities, and tuition fees are barely made up for by low-paying jobs, while high-paying jobs (e.g. investment banking) pay significantly more than tuition costs.

The intuition for why tuition pricing is a dominant strategy under this model is that ISAs suffer from *adverse selection.* When given a choice between ISAs and tuition fees, high-type students will opt out of ISAs because they have higher future wages, so it is cheaper for them to pay tuition. Conversely, low-type students will opt into ISAs because they have lower future wages, so it is cheaper for them to take an ISA than to pay potentially unaffordable tuition fees. The result is that universities that adopt ISAs can only attract low-type students, who pay lower fees and thus are less profitable. Thus, tuition fees are more profitable in this environment. The problem of adverse selection is likely to persist even when moving beyond a duopoly into a general oligopoly or a competitive market, and offers key intuition for the problems that ISAs may face in gaining traction.

Finally, unlike the monopoly outcome, the duopoly equilibrium is socially efficient. Competition eliminates the social inefficiency caused by universities artificially restricting supply under tuition pricing, but it fails to eliminate the social inefficiency caused by ISAs decreasing student investment in productivity. Thus, tuition pricing maximizes both the supply of education as well as the productivity of education. Even though this stylized fact is established in a duopoly setting, the effect of competition would only grow more stark under more general competitive models. This fact suggests that policymakers should not shift towards adopting ISAs more broadly.

The rest of the paper is organized as follows. Section 2 explores the related literature and highlights the original contributions made by this paper. Section 3 provides the main model of a monopolist university, and introduces the analysis under a tuition pricing model. Section 4 introduces ISA pricing to this model and analyzes the new optimal mechanisms, finally comparing their social welfare to that achieved under tuition pricing. Section 5 generalizes this analysis to a duopoly setting, and analyzes the two-stage game of choosing a pricing model and finding equilibrium under each pricing model. Section 6 concludes this analysis and discusses directions for future research.

2. Literature Review

This paper draws on two parallel and often conflicting streams of literature in education: job market signalling, and human capital theory. The basic model presented is an extension of job market signalling (Spence 1973), in which education is a way for individuals to signal their inherent ability even though there is no value to the education itself. This basic model has been extended to many different educational scenarios, such as the decision to make and take early job offers (Swinkels 1999), the effect of specific grades on market signalling (Daley and Green 2014), the "sheepskin effect" of a degree in enhancing the earnings of workers with equivalent real education levels to non-degree workers (Belman and Heywood 1991, Heywood 1994, Jaeger and Page 1996) and even the decision not to pursue education (Orzach and Tauman 2005). The contribution of this paper to the signalling literature is twofold: first, we apply signalling theory to income-share agreements to generate new insights. Second, almost all papers on signalling ignore the empirical reality that the university is itself a profit-maximizing agent who determines the cost of education. This fact is incorporated into our model, resulting in a richer analysis of signalling behavior in practice.

Most of the signalling literature disregards the university as a strategic agent. One exception is the literature on how universities strategically disclose information about students to employers: for example, Ostrovsky and Schwarz (2010) study a university's information disclosure policies, and find that in equilibrium, universities disclose a balanced amount of information on students to the labor market. The key feature of this information disclosure literature is that it takes the relation between students and universities as given, and analyze strategic behavior between universities and the labor market. In contrast, this paper analyzes strategic behavior between students and universities, while taking the labor market as given. Thus, it complements the literature on information disclosure and lays the groundwork for a more complete model that incorporates the capacity for a university to behave strategically in both spheres.

A key feature of this model in this paper, and another departure from the signalling literature as a whole, is that student productivity is a function of costly private effort. This feature of the model is grounded in Becker's human capital theory of education, in which education serves to increase an individual's productivity.(Becker 1962) Human capital theory and signalling are often framed as competing explanations for why higher education increases wages. (Layard and Psacharopoulos 1974, Wise 1975, Chevalier et al. 2004, Weiss 1995) Our incorporation of productivity and effort into a signalling model sidesteps this debate, allowing for a more flexible model that can incorporate both the signalling effects of education as well as its productivity-enhancing characteristics.

Finally, this paper is related to the small body of work that directly studies income-share agreements. The only other economic analysis of ISAs that we are aware of comes from Madonia and Smith (2019), who find that when poker players finance their tournament entries with ISAs, they tend to earn less. Madonia and Smith attribute 80% of this performance gap to diminished individual incentives to perform, supporting our qualitative result that under ISAs, students invest significantly less in their own human capital. Furthermore, ISAs have received some attention in policy spheres: Oei and Ring (2015) offer a legal framework for regulating ISAs, while Palacios, DeSorrento, and Kelly (2014) argue in a policy brief for the US government to provide ISAs to students, as an alternative to federal student loans. Such work is important in formulating a policy approach to ISAs, and this paper contributes a policy-relevant framework within which the effect of ISAs can be examined. Even if this model rests on assumptions that may not hold in all cases, the qualitative insights it generates—ISAs reduce student investment, and suffer from adverse selection—are both empirically testable, as well as important considerations for policymakers.

3. Tuition Pricing

Consider a profit-maximizing university with a fixed supply of seats, normalized to 1. The university faces a population of students of size $N > 1$. Students can be either a high type or a low type, $t \in \{H, L\}$. The types differ by their future wages, $w_H > w_L$. The population of each type is N_t, with $N_H + N_L = N > 1$. For the rest of the paper we will assume $N_H, N_L < 1$—that is, the population is relatively balanced such that neither one could feasibly take up all slots in the university.

The university provides a productive education that can *convert* a low-type to a high type with some probability $\phi(e)$ as a function of the student's costly effort level e. High-types do not benefit from exerting effort: to them, the value of the university education is purely signalling that they are high-types on the labor market. We assume that the productivity function ϕ satisfies

$$\phi(0) = 0,\ \phi'(e) > 0,\ \lim_{e \longrightarrow \infty} \phi(e) = 1$$

In words, a student who exerts zero effort does not change their type, and a student strictly increases their probability of becoming a high-type by exerting more effort. For the rest of the paper, we adopt the functional form

$$\phi(e) = \frac{e}{e+1}$$

although the results are true for all functional forms unless noted otherwise.

Students choose their effort level to maximize their private benefit. Exerting effort e costs a student $k(e)$ where k is some strictly increasing and convex cost function. The specific form of $k(e)$ does not matter to all results in this paper. This cost is mirrored by the potential type improvement, given by $\phi(e)$. This productivity of education distinguishes a student's ex-ante type (before entering university) from a student's ex-post type (after exerting effort through the education technology). The university truthfully reports a student's ex-post type to the labor market. Each ex-post type on the labor market receives competitive wage w_t with $w_H > w_L$. For ease of notation, we denote the expected wage of the low-type by

$$W(e) = \phi(e)w_H + (1 - \phi(e))w_L$$

Let p_t denote the university's probability of admitting a student who declares their type to be t, and c_t be the tuition fee charged to each declared type. Then we can define an important concept used for the rest of the paper.

Definition 1. *A* ***mechanism*** *is a admissions policy* (p_H, p_L, c_H, c_L) *chosen by the university.*

In maximizing its profit, the university chooses a mechanism that solves the problem

$$
\begin{aligned}
\max_{p_H,p_L,c_H,c_L} \quad & p_H N_H(c_H - ke_H) + p_L N_L(c_L - ke_L) && \text{(Obj)}\\
\text{s.t.} \quad & p_H(w_H - c_H - k(e_H)) \geq p_L(w_H - c_L - k(e_L)) && \text{(IC-H)}\\
& p_L(W(e_L) - c_L - k(e_L)) \geq p_H(W(e_H) - c_H - k(e_H)) && \text{(IC-L)}\\
& p_H(w_H - c_H - k(e_H)) \geq 0 && \text{(IR-H)}\\
& p_L(W(e_L) - c_L - k(e_L)) \geq 0 && \text{(IR-L)}\\
& p_H N_H + p_L N_L \leq 1 && \text{(Supply)}
\end{aligned}
$$

Each constraint represents a necessity for truthful participation by all students. (IC-H) represents the constraint that high-types can never achieve higher surplus by claiming to be low-types. This misreport would allow them admission with probability p_L and to pay the fee c_L, but they would nevertheless exert the true high-type effort e_H and get wage w_H because the university does not control those factors. Likewise, (IC-L) represents the constraint that low-types can never achieve higher surplus by claiming to be high-types and facing (p_H, c_H) while exerting effort e_L and getting wage $W(e_L)$. (IR-H) and (IR-L) represent the constraints that high-types and low-types must receive nonnegative surplus conditional on attending the university, or else they would not show up to the university to begin with. The combined tuition fee and private effort cost must not exceed the wage benefit. Note that the outside option (no college) is normalized to have utility 0. Finally, (Supply) reflects the simple constraint that the university has a limited supply of seats (e.g. from limited campus space and infrastructure) and cannot allocate more seats than that.

Note that these constraints collectively imply that the distribution of types reported to the university is always the same: all students show up to the university and report their true type. This means that we can define the mechanism without explicitly referencing the distribution of reported types, only the type of a given student reporting, which simplifies the analysis. Appendix A.1 proves that this is without loss of generality.

3.1. Interior-Optimal Mechanisms

Consider the student's optimization problem.

Proposition 1. *When students respond optimally to a mechanism,* $e_H^* = 0$ *and* $e_L^* > 0$.

Proof. For a high type student, education is unproductive, and they can report their true high type without exerting any effort. Thus, $e_H^* = 0$ is optimal. For a low-type student, this is not the case. The private benefit for a low-type from exerting effort is

$$u(e) = W(e) - c_L - k(e)$$

First, assume an interior solution with $e_L^* > 0$: then taking the FOC yields

$$u'(e) = W'(e) - k'(e) = 0$$

$$\implies \frac{(e+1)w_H - ew_H - w_L}{(e+1)^2} = k'(e)$$

$$\implies w_H - w_L = k'(e_L^*)(e_L^* + 1)^2$$

Note that since k is strictly increasing, this has a strictly positive solution in e: the function

$$f(e) = k'(e)(e+1)^2$$

satisfies

$$f(0) > 0, f(e_L^*) = w_H - w_L > 0$$

and is strictly increasing and continuous on $\mathbb{R}_+$. Thus, by the intermediate value theorem, the preimage of each $y > 0$ is some $x > 0$. Thus, the preimage of

$$f(e_L^*) = k'(e_L^*)(eL^* + 1)^2$$

is some $e_L^* > 0$. □

Proposition 1 does not give a concrete value for the optimal e_L^* because that depends on the functional form of $k(e)$. However, the functional form of e_L^* is not necessary to the results that follow: all that matters is that $e_H^* = 0, e_L^* > 0$. Furthermore, since the optimal effort level is independent of p_t, c_t it does not vary in the incentive/participation constraints—in a sense, the student's choice is the same regardless of what mechanism the university chooses, since the effort is exerted only after admission and is additively separable from the cost c_t. This changes the university's problem to

$$
\begin{aligned}
\max_{p_H,p_L,c_H,c_L} \quad & p_H N_H c_H + p_L N_L c_L && \text{(Obj)}\\
\text{s.t.} \quad & p_H(w_H - c_H) \geq p_L(w_H - c_L) && \text{(IC-H)}\\
& p_L(W(e_L) - c_L - k(e_L)) \geq p_H(W(e_L) - c_H - k(e_L)) && \text{(IC-L)}\\
& p_H(w_H - c_H) \geq 0 && \text{(IR-H)}\\
& p_L(W(e_L) - c_L - k(e_L)) \geq 0 && \text{(IR-L)}\\
& p_H N_H + p_L N_L \leq 1 && \text{(Supply)}
\end{aligned}
$$

It is clear that in any profit-maximizing allocation, $c_H, c_L > 0$. Furthermore, this section considers only interior solutions $p_H, p_L > 0$—this assumption will be relaxed in the next section. To characterize the optimal mechanism, we will use the following lemma.

Lemma 1. *In an interior optimum, (IC-H), (IR-L) and (Supply) constraints bind.*

Proof. See Appendix B. □

Then we can pin down the unique interior optimal mechanism.

Proposition 2. *The unique interior optimal mechanism is*

$$p_H = 1, p_L = \frac{1 - N_H}{N_L}, c_L = W(e_L) - k(e_L), c_H = \frac{(N-1)w_H - (1 - N_H)(W(e_L) - k(e_L))}{N_L}$$

Proof. Since (IR-L) binds by Lemma 1, we immediately have

$$c_L = W(e_L) - k(e_L)$$

Since (IC-H) binds,

$$
\begin{aligned}
& p_H(w_H - c_H) = p_L(w_H - c_L)\\
\implies & w_H - c_H = \frac{p_L}{p_H}(w_H - W(e_L) + k(e_L))\\
\implies & c_H = w_H - \frac{p_L}{p_H}(w_H - W(e_L) + k(e_L))
\end{aligned}
$$

Furthermore, maximizing c_H requires minimizing $\frac{p_L}{p_H}$. Lemma 1 also gives us that

$$p_L = \frac{1 - p_H N_H}{N_L}$$

so this minimization corresponds to

$$p_H = 1, p_L = \frac{1 - N_H}{N_L}$$

This yields a final value of c_H as

$$c_H = \frac{(N-1)w_H - (1 - N_H)(W(e_L) - k(e_L))}{N_L}$$

Then the interior optimal mechanism has been pinned down exactly as

$$p_H = 1, p_L = \frac{1 - N_H}{N_L}, c_L = W(e_L) - k(e_L), c_H = \frac{(N-1)w_H - (1 - N_H)(W(e_L) - k(e_L))}{N_L}$$

□

This mechanism is straightforward to characterize. The university admits all high-type students, fills the remaining seats with low-type students, and charges low-type students their full wage. The high-type fee is set at the highest level possible that respects (IC-H) given the low-type fee and the admissions rates.

3.2. Shut-Out Mechanisms

Until now, we assumed $p_H, p_L > 0$. However, in an extreme scenario, it may be profitable for the university to "shut out" one type by imposing $p_L = 0$ or $p_H = 0$, in order to extract higher surplus from the other type. In this section we analyze when such shut-outs can be optimal. First, consider

$$p_H = 1, p_L = 0$$

Then (IC-L) and (IR-L) become vacuous, and (IC-H) becomes identical to (IR-H). Thus, the only constraint is

$$p_H(w_H - c_H) \geq 0$$

, and it is clearly optimal to make this bind. Thus,

$$c_H = w_H$$

The resulting profit is

$$\pi = N_H w_H$$

The other shut-out mechanism satisfies

$$p_H = 0, p_L = 1$$

However, this violates monotonicity: it cannot be incentive-compatible because H-types can always claim to be L-types, unless

$$c_L > w_H$$

$$\implies c_L > W(e_L) - k(e_L)$$

in which case (IR-L) is violated. We will henceforth use “shut-out mechanism” to reference a mechanism that shuts out low-types.

Proposition 3. *The shut-out mechanism is optimal iff*

$$N_H w_H \geq (N_L - N_H)(W(e_L) - k(e_L))$$

Proof. The profit from the interior optimum is

$$\pi = \frac{N_H}{N_L}(N-1)w_H + (1-N_H)(\frac{N_L - N_H}{N_L})(W(e_L) - k(e_L))$$

whereas the profit from the shut-out mechanism is

$$\pi = N_H w_H$$

Thus the comparison is

$$N_H w_H \geq \frac{N_H}{N_L}(N-1)w_H + (1-N_H)(\frac{N_L - N_H}{N_L})(W(e_L) - k(e_L))$$

$$\implies w_H \geq \frac{1}{N_L}(N-1)w_H + (1-N_H)(\frac{1}{N_H} - \frac{1}{N_L})(W(e_L) - k(e_L))$$

$$\implies w_H(\frac{1-N_H}{N_L}) \geq (1-N_H)(\frac{1}{N_H} - \frac{1}{N_L})(W(e_L) - k(e_L))$$

$$\implies w_H \geq (\frac{N_L - N_H}{N_H})(W(e_L) - k(e_L))$$

$$\implies N_H w_H \geq (N_L - N_H)(W(e_L) - k(e_L))$$

□

Corollary 1. *If $N_H > N_L$, the shut-out mechanism is optimal.*

Proof. If we have $N_L - N_H < 0, W(e_L) - k(e_L) \geq 0$ then the RHS of the inequality in Proposition 3 is negative while the LHS is strictly positive, so the inequality holds. □

In mechanism design, it is well-known that revenue maximization may often involve setting reserve prices: prices below which the item is never sold regardless of participant valuations. Shut-out mechanisms are the natural extension of reserve prices to the binary-type model presented here: the reserve price is placed at w_H and only people who can pay that much (true high-types) will accept the mechanism.

4. Income-Share Agreements

Income-share agreements have grown in prominence as a funding mechanism for tertiary education. In essence, under an ISA model, students pay a fraction of their income rather than a fixed fee. In this model, there are only two wages, and they are static and completely determined by an agent's ex-post type in the labor market. Thus, ISAs would correspond to the price of admission being conditioned on ex-post type rather than announced ex-ante type. In this section, we consider a model where the monopolist university follows ISA pricing rather than tuition pricing.

Under ISA pricing, the university specifies a probability of admission p_t for each announced type, and a fraction of income demanded from all students α. In particular, α is defined independently of student type so that we effectively have

$$c_H = \alpha w_H, c_L = \alpha W(e_L)$$

This changes the university's problem from the tuition case to

$$\max_{\alpha, p_H, p_L} \; p_H N_H \alpha w_H + p_L N_L \alpha W(e_L)$$

$$\text{s.t. } p_H(1-\alpha)w_H \geq p_L(1-\alpha)w_H$$

$$p_L((1-\alpha)W(e_L) - k(e_L)) \geq p_H((1-\alpha)W(e_L) - k(e_L))$$

$$p_H(1-\alpha)w_H \geq 0$$

$$p_L((1-\alpha)W(e_L) - k(e_L)) \geq 0$$

$$p_H N_H + p_L N_L \leq 1$$

4.1. ISA Optimum

Under ISA pricing, unlike under tuition pricing, there is a unique optimal mechanism, which is simple to characterize.

Proposition 4. *Under ISA pricing, the unique optimal mechanism is*

$$\alpha = 1, p_H = 1, p_L = \frac{1 - N_H}{N_L}$$

Proof. Note that $\alpha = 1$ vacuously satisfies all IC and IR constraints, because both LHS and RHS reduce to 0. This is more subtle for (IR-L), because low-types who incur private effort costs that aren't extracted by an ISA. However, it is still true: an L-type student who faces $\alpha = 1$ can always choose $e_L = 0$ to be optimal, which results in nonnegative surplus. This changing nature of effort will be expanded upon further in section 4.2. Furthermore, $\alpha = 1$ maximizes profit among all values of $\alpha \in [0, 1]$ because the profit function is strictly increasing in α: $\frac{\partial \pi}{\partial \alpha} = p_H N_H w_H + p_L N_L W(e_L) > 0$ if $\min\{p_H, p_L\} > 0$ which must be true in a profit-maximizing mechanism.

Thus, the problem can be reframed as

$$\max_{p_H, p_L} \; p_H N_H w_H + p_L N_L w_L$$

$$\text{s.t. } p_H N_H + p_L N_L \leq 1$$

The supply constraint clearly binds and so $p_L N_L = 1 - p_H N_H$ so the profit is simply

$$\pi(p_H) = p_H N_H W_H + w_L - p_H N_H w_L$$

$$\implies \pi(p_H) = p_H N_H (w_H - w_L) + w_L$$

This is strictly increasing in p_H because $w_H > w_L$ so it is optimal to set $p_H = 1$, the highest possible value. Thus, the unique optimal mechanism under an ISA system is given by

$$\alpha = 1, p_H = 1, p_L = \frac{1 - N_H}{N_L}$$

□

Corollary 2. *Under ISA pricing, the university never adopts a shut-out mechanism.*

The mechanism in Proposition 4 resembles the tuition-based university's interior optimal mechanism, in the sense that all high-types are admitted and the remaining seats are distributed among low-types. However, it achieves significantly higher rent extraction: the tuition-based university cannot extract full wage from high-types without violating their incentive constraints, but the contingent nature of ISAs makes incentive constraints vacuous. As a result, there is never any benefit from shutting out low-types, because it cannot increase rent extraction from high-types.

4.2. Optimal Effort Under ISAs

Though ISAs reduce shut-outs, they can hurt one aspect of education that is generally considered to be very important: its capacity to improve people's productivity. Student productivity is determined by their effort level, and the following result demonstrates that the optimal effort level for students is reduced under an ISA model.

Proposition 5. *Under an ISA at the level α, the optimal effort level for low-types is strictly decreasing in α. When $\alpha = 1$, the optimal effort is $e_L^* = 0$.*

Proof. When $\alpha = 1$, students gain zero surplus from the labor market and internalize the costs of effort. This means that any $e_L > 0$ leads to strictly negative utility, whereas $e_L = 0$ leads to zero utility. Thus, it is optimal to set $e_L^* = 0$.

Under an ISA, the student's problem is

$$\max_{e_L}(1-\alpha)W(e_L) - k(e_L)$$

$$\implies (1-\alpha)W'(e_L) - 2k'(e_L) \leq 0$$

Assume an interior solution, and recall that

$$W(e_L) = (\frac{e_L}{e_L+1})(w_H - w_L) + w_L$$

$$\implies W'(e_L) = \frac{w_H - w_L}{(e_L+1)^2}$$

$$\implies (1-\alpha)(w_H - w_L) = k'(e_L)(e_L+1)^2$$

This is satisfied by any optimal e_L^*. We can implicitly differentiate this with respect to α to get

$$-w_H + w_L = k''(e) \cdot e'(\alpha) \cdot (e+1)^2 + e'(\alpha) \cdot 2k'(e)(e+1)$$

$$\implies e_L'(\alpha)[k''(e)(e+1)^2 + 2k'(e)(e+1)] = w_L - w_H$$

$$\implies e'(\alpha) = \frac{w_L - w_H}{k''(e) \cdot (e+1)^2 + 2k'(e) \cdot (e+1)}$$

This numerator is strictly negative ($w_H > w_L$) and the denominator is strictly positive ($k' > 0, k'' \geq 0, e_L \geq 0$) so this derivative is strictly negative. Thus, effort is strictly decreasing in α. □

Although ISAs can prevent shut-outs, they hurt the aspect of education that is often thought to be the most important: its productivity-enhancing characteristics. Under an ISA model, students reduce their investment in their own human capital below the optimal level in a tuition-based model.

4.3. Social Welfare Under ISAs

When comparing social welfare under the ISA and tuition models, we can differentiate between two quantities of interest:

1. The welfare achieved by the university's profit-maximizing mechanism under each model

2. The welfare achieved by the social optimum under each model

Proposition 6 demonstrates the first comparison.

Proposition 6. *Let r_T, r_I be the profit maximizing mechanisms under tuition pricing and ISA pricing respectively. Then*

$$U(r_I) > U(r_T) \iff N_H w_H > N_L(W(e_L) - k(e_L))$$

Proof. ($\impliedby$) : When $N_H w_H > N_L(W(e_L) - k(e_L))$, by Proposition 3 r_T is a shut-out mechanism, which achieves $U(r_T) = N_H w_H$. The welfare of r_I is

$$U(r_I) = 1 \cdot N_H w_H + (\frac{1 - N_H}{N_L}) \cdot N_L w_L$$

$$\implies U(r_I) = U(r_T) + (1 - N_H) w_L$$

$$\implies U(r_I) > U(r_T)$$

($\implies$) : When $N_H w_H < N_L(W(e_L) - k(e_L))$, r_T satisfies $p_H = 1, p_L = \frac{1-N_H}{N_L}$ and thus

$$U(r_T) = 1 \cdot N_H w_H + (\frac{1 - N_H}{N_L}) \cdot N_L W(e_L)$$

$$\implies U(r_T) = N_H w_H + (1 - N_H) W(e_L)$$

$$\implies U(r_T) > N_H w_H + (1 - N_H) w_L$$

$$\implies U(r_T) > U(r_I)$$

□

We can analyze Proposition 6 more. The scenario dependence comes from the polar opposite nature of the mechanisms chosen by the university under a tuition model. When $N_H w_H > N_L(W(e_L) - k(e_L))$, the comparison is to a shut-out mechanism which is clearly wasteful. However, when $N_H w_H < N_L(W(e_L) - k(e_L))$ the tuition model incentivizes the university to allocate all slots in the same way as the ISA model: the only difference is that the ISA model disincentivizes student investment in their own productivity ($e_L^* = 0$) which leads to lower social welfare.

5. Duopoly

This section generalizes the model beyond the case where the university is a monopolist. Let there be two identical universities A, B, each with capacity 1. The significant change in the environment is that all students apply to both universities and receive offers with independent probabilities. Importantly, the universities cannot coordinate their offers: for example, if $p_H^A = p_H^B = 0.5$, the universities cannot coordinate such that half of H-types are admitted solely to A and the other half are admitted solely to B. Formally, a student who announces type $t \in \{H, L\}$ is admitted to only A with probability $p_t^A(1-p_t^B)$, only B with probability $p_t^B(1-p_t^A)$, to both universities with probability $p_t^A p_t^B$, and to neither university with probability $(1 - p_t^A)(1 - p_t^B)$. Students who are admitted to both universities choose the university where they will pay a lower expected price, whether that is in the form of tuition fees or income-shares.

As a consequence of competition, a university's intake of students is also a function of its competitor's admissions policy. To prevent violations of the supply constraint, the university is not allowed to overallocate seats: the supply constraint remains unchanged at

$$p_L^u N_L + p_H^u N_H \leq 1, u \in \{A, B\}$$

Furthermore, in the monopoly case we assumed $N > 1$ to capture the the idea that the university could not service all demand. In this duopoly environment we extend this to assume that $N_t > 1 \; \forall \, t \in \{H, L\}$: in other words, no university can serve all demand from each type.

In this environment, a major question of interest is—what model is endogenously adopted by universities who seek to maximize their equilibrium profit? And is this outcome efficient? In other words, we now consider a two-stage game: in the first stage, universities simultaneously choose a pricing model from $\{\text{Tuition}, \text{ISA}\}$. In the second stage, universities simultaneously choose the optimal mechanism given their pricing model and their competitor's pricing model. To evaluate this dynamic game, we use the concept of a subgame-perfect equilibrium, a refinement of Nash equilibrium.

Definition 2. *A strategy profile is a* ***subgame-perfect equilibrium*** *if it induces universities to play a Nash equilibrium in every contingent scenario of the game.*

In this dynamic game, a strategy is a subgame-perfect equilibrium if it prescribes a Nash equilibrium in every possible duopoly—namely, a tuition-tuition duopoly, a tuition-ISA duopoly, and an ISA-ISA duopoly. We study each of these scenarios in turn, before evaluating the entire game.

5.1. Tuition Duopoly

Consider first the duopoly where two universities charge tuition fees to students. Under the duopoly market, a particular class of mechanism profiles becomes worth studying.

Definition 3. *A* ***sorting profile*** *is a pair of mechanisms such that*

$$p_t^A > 0 \iff p_t^B = 0, p_t^B > 0 \iff p_t^A = 0, c_t^A = c_{t'}^B = c$$

where t is the type admitted by A, and t' is the type admitted by B.

In other words, a sorting profile is one where each university selects a single type to admit, and all types pay the same fee. Sorting profiles turn out to be vital in characterizing pure equilibria in the duopoly market.

Proposition 7. *A sorting profile with $c = W(e_L) - k(e_L)$ for $t \neq t'$ is a pure equilibrium in the tuition duopoly.*

Proof. First, we demonstrate a sorting equilibrium. Consider the sorting profile where A admits L and B admits H, with

$$p_L A = \frac{1}{N_L}, p_H^B = \frac{1}{N_H}, p_H^A = p_L^B = 0, c_L^A = c_H^B = W(e_L) - k(e_L)$$

This profile is an equilibrium. To see this, first note that neither university can raise fees on the type they admit. Raising fees would violate the participation constraint for L-types, and would violate the incentive constraints for H-types: they would simply lie about being L-types and attend the other university at the fee $W(e_L) - k(e_L)$. Second, note that neither university can benefit from lowering fees on the type they admit: they have fully allocated their seats since $N_t \geq 1 \implies p_t N_t = 1$, so they have no capacity to serve excess demand by reducing fees. Third, note that neither university can strictly improve by changing their admissions rates p_H^A, p_L^B without changing tuition fees, because *every* student pays the same fee $W(e_L) - k(e_L)$ so any combination of types admitted yields the same profit.

This leaves the only possible deviation as one where a university changes both admissions policies and tuition fees. The most profitable such deviation is $p_H^A = \frac{1}{N_H}, p_L^A = 0, c_H^A = w_H$. $c_H^A = w_H$ is the uniquely best deviation in fees because any $c_H^A \leq W(e_L) - k(e_L)$ cannot raise profit over the existing $W(e_L) - k(e_L)$ while all $c_H^A > W(e_L) - k(e_L)$ attract the same number of students, so it is always profitable to make it as high as possible. The profit from this deviation is $\frac{1}{N_H}(1 - \frac{1}{p_H^B}) N_H w_H = w_H(1 - \frac{1}{N_H})$

Thus, this sorting profile is an equilibrium. □

Furthermore, Proposition 12 shows that this sorting profile turns is the *unique* pure equilibrium in the tuition duopoly. Thus, in the tuition duopoly, pure equilibrium is very simple to characterize: both universities charge a flat fee, and that fee is set at the full wage of low-types, the highest amount they can pay.

5.2. Tuition-ISA Duopoly

Consider now a duopoly where university T charges tuition fees c_H, c_L, whereas university I charges income-share α. The model is otherwise identical to the tuition-tuition duopoly: in particular, low-type students evaluating university I choose their effort level based on the value of α, and choose the university based on

$$(1-\alpha)W(e_L(\alpha)) - k(e_L(\alpha)) \gtreqless W(e_L) - k(e_L) - c_L$$

Here, too, the unique pure equilibrium is a sorting profile.

Proposition 8. *The profile with*

$$p_H^T = \frac{1}{N_H}, p_L^I = \frac{1}{N_L}, c_H^T = w_H, \alpha = 1$$

is an equilibrium in the tuition-ISA duopoly.

Unlike in the tuition duopoly, this equilibrium cannot be reordered: in particular, it is not an equilibrium for T to take low-types and I to take high-types. This arises because high-types have an incentive to select out of ISAs and into tuition when tuition is set at the rate of low-type students, as that gives them strictly greater surplus than an ISA that extracts their full surplus. Thus, ISAs display adverse selection: in equilibrium, only low-type students attend ISAs.

5.3. ISA Duopoly

Consider a duopoly where two universities A, B offer income-shares α^A, α^B. One useful fact about this setting is that a sorting profile is feasible if and only if $\alpha^A = \alpha^B = 1$. This is proved in Appendix C. Unlike the other two subgames, the ISA duopoly subgame is not amenable to pure equilibrium analysis.

Proposition 9. *No pure equilibrium exists in an ISA duopoly.*

Proof. Consider the candidate equilibrium profiles:

1. A sorting profile. Note that $\alpha^A = \alpha^B = 1$ is a feasibility requirement by Lemma 5 and neither university can feasibly deviate on price alone. In order to deviate, a university has to choose $\alpha < 1$ and also set $p_H = p_L = p$. The best such deviation is by the university that takes low-types, say A, and sets $p^A = \frac{1}{N_H+N_L}, \alpha^A = 1-\epsilon$. This is profitable if

$$\frac{N_H(1-\epsilon)w_H + N_L(1-\epsilon)W(\alpha^A)}{N_H + N_L} > w_L$$

$$\implies 1-\epsilon > \frac{N_H w_L + N_L w_L}{N_H w_H + N_L W(\alpha^A)}$$

$$\implies \epsilon < \frac{N_H w_H + N_L W(\alpha^A) - N_H w_L - N_L w_L}{N_H w_H + N_L W(\alpha^A)}$$

$$\implies \epsilon < \frac{N_H(w_H - w_L) + N_L(W(\alpha^A) - w_L)}{N_H w_H + N_L W(\alpha^A)}$$

   Since both the numerator and denominator are strictly positive, this is always a feasible deviation—so no sorting profile can be an equilibrium.

2. At least one type is admitted by both universities with positive probability. Then B can change to $\alpha' = \frac{\alpha^A+\alpha^B}{2} > \alpha^B$ which is a strict improvement: it

	Tuition	ISA
Tuition	(W-k)(e_L), (W-k)(e_L)	w_H, w_L
ISA	w_L, w_H	π_A^I, π_B^I

Table 1: Choosing a pricing model under duopoly.

Payoffs are equilibrium profits to each university under each subgame.

increases revenue while not losing students of either type to A, regardless of what p_t^A, p_t^B actually are. Thus, no such profile can be an equilibrium.

□

Thus, any equilibrium in the ISA duopoly must be a mixed strategy equilibrium. However, characterizing a mixed equilibrium requires very strong assumptions on $k(e), w_H, w_L$. For the purpose of our two overarching questions—what is the subgame-perfect equilibrium, and what is the socially optimal outcome?—characterizing a mixed equilibrium is not necessary.

5.4. Duopoly Subgame-Perfect Equilibrium

With the three subgames analyzed, we can analyze the SPE of the duopoly game, and separately analyze what the most efficient outcome might be. By definition of an SPE, it requires that in each duopoly, the universities play an equilibrium strategy. This is equivalent to analyzing a 2x2 matrix game with the equilibrium payoffs from each subgame Nash equilibrium. Table 1 shows this payoff matrix, with undetermined profits under the ISA duopoly because we have not characterized the mixed equilibrium.

Proposition 10. *It is a dominant strategy for universities to adopt a tuition model.*

Proof. First, note that

$$W(e_L^*) - k(e_L^*) > w_L$$

This follows from the fact that

$$w_L = W(0) - k(0)$$

and $e_L = 0$ was a feasible solution, but we know $e_L^* > 0$ is the unique maximizer from Proposition 1. Second, note that in any ISA mixed equilibrium,

$$\pi_A^I < w_H, \pi_B^I < w_H$$

because a profit of w_H follows only from the non-degenerate mechanism of filling all seats with high-type students and charging them their full wage. Any mechanism that admits low-types with positive probability (as both universities must do in an ISA mixed equilibrium, by Lemma 5) cannot charge w_H to low-type students (that would violate their participation constraints) and thus cannot receive w_H from all students. This results in

$$\pi_A^I < w_H, \pi_B^I < w_H$$

Since

$$(W - k)(e_L^*) > w_L, w_H > \max(\pi_A^I, \pi_B^I)$$

we can conclude that Tuition is a dominant strategy for both universities in the game of Table 1. □

Corollary 3. *The unique SPE of the duopoly model is for both universities to adopt a tuition model and enter a sorting equilibrium with*

$$p_t^A = \frac{1}{N_t}, p_{t'}^B = \frac{1}{N_{t'}}, c_t^A = c_{t'}^B = W(e_L) - k(e_L)$$

The dominance of tuition over ISAs is surprising given that in the monopoly environment, ISAs achieved the highest possible profit. We can deconstruct this result to understand why this profitability reverses under the duopoly. The first factor driving this reversal is adverse selection. Under a tuition-ISA duopoly, an ISA university cannot attract high-types and receive higher profits from them, because the tuition university must take low-types and charge the low-type wage, at which price it is not incentive-compatible for high-types to attend the ISA university. Thus, in equilibrium only low-types attend the ISA university, while high-types attend the tuition university and are charged their full wage. Thus, adverse selection makes it unprofitable to deviate from charging tuition to charging ISAs.

The second factor driving this reversal, which is also very relevant to analyze the efficiency of each outcome, is that competition forces all seats to be allocated, regardless of pricing model: a shut-out mechanism is simply not optimal when a competitor can undercut you. Thus, the allocation of seats is unchanged between tuition and ISA models. However, in the tuition-ISA equilibrium, the ISA university charges full wage $\alpha = 1$: so low-type students do not exert any effort, and the maximum rent that can be extracted from them is w_L. Thus, tuition pricing allows for the surplus from student effort to be extracted by the university whereas ISA pricing does not.

Thus, the key inefficiency of the tuition model—a tuition-based monopolist doesn't allocate all seats—disappears under competition, but the key inefficiency of the ISA model—an ISA-based monopolist disincentivizes all effort—remains under competition. As a result, tuition pricing is socially superior to ISA pricing in both productivity of education and the availability of education, and the tuition duopoly is efficient.

6. Conclusion

In this paper, we have studied the implications of adopting income-share agreements over tuition fees for the efficiency of education. When the university is a monopolist, ISA pricing incentivizes the university to increase the supply of education and drop socially inefficient reserve prices. However, it also reduces student surplus and disincentivizes students from investing in their own productivity. In this way, ISAs have a mixed effect on education efficiency, and whether they are socially superior depends on the relative value a policymaker places on the productivity of an education and the availability of education. Under a duopoly, this result is starker: tuition maximizes both the supply of education and the incentives of students to invest, making it more efficient than ISA pricing. Furthermore, it is a dominant strategy for universities to choose tuition pricing over ISA pricing, so the unique equilibrium under the duopoly game is also efficient. A key factor in this dominance is that ISAs display adverse selection: in a tuition-ISA equilibrium, ISAs are only chosen by low-type students, making them less profitable than tuition fees.

The model presented in this paper is only a starting point for work on the phenomenon of income-share agreements. In particular, future work can develop richer models in which students may have to take on debt in order to attend a tuition-based university. This incorporation could address more directly the policy debate over ISAs, which often relates to concerns over student debt, and deepen the comparison between tuition and ISAs beyond that which is explored here.

Future work could also expand the model to consider cases where human capital is jointly produced by the university and the student, rather than solely a product of the student effort. Proponents of ISAs often suggest that one beneficial effect of ISAs would be to increase university incentives to invest in student productivity. Given our results about decreasing student incentives to invest, there is much scope to analyze the possible complementarities between university effort and student effort, and whether these opposing directions of change create a net increase or decrease in student productivity.

Finally, with growing adoption of ISAs, the scope to study them empirically is growing as well. This paper offers theoretical predictions that can be clearly tested in an empirical setting: it predicts that ISAs would display adverse selection, attracting primarily high-type students, and that ISAs would decrease student investment. These predictions offer a basis for empirical work to form hypotheses about ISAs, and such empirical work would be highly complementary to the goal of understanding ISAs and better understanding how they affect the education system.

References

Becker, Gary S. 1962. “Investment in human capital: A theoretical analysis”. *Journal of political economy* 70 (5, Part 2): 9–49.

Belman, Dale, and John S Heywood. 1991. “Sheepskin effects in the returns to education: An examination of women and minorities”. *The Review of Economics and Statistics*: 720–724.

Chevalier, Arnaud, Colm Harmon, Ian Walker, and Yu Zhu. 2004. “Does education raise productivity, or just reflect it?” *The Economic Journal* 114 (499): F499–F517.

Daley, Brendan, and Brett Green. 2014. “Market signaling with grades”. *Journal of Economic Theory* 151:114–145.

Friedman, Milton. 1955. *The role of government in education.*

FT. 2019. “Shares in students: nifty finance or indentured servitude?” Visited on 04/21/2020. https://www.ft.com/content/46827e2a-f990-11e9-a354-36acbbb0d9b6.

Heywood, John S. 1994. “How widespread are sheepskin returns to education in the US?” *Economics of Education Review* 13 (3): 227–234.

Jaeger, David A, and Marianne E Page. 1996. “Degrees matter: New evidence on sheepskin effects in the returns to education”. *The review of economics and statistics*: 733–740.

Kirshstein, Rita J. 2012. “Not Your Mother’s College Affordability Crisis. Issue Brief.” *Delta Cost Project at American Institutes for Research.*

Layard, Richard, and George Psacharopoulos. 1974. “The screening hypothesis and the returns to education”. *Journal of political economy* 82 (5): 985–998.

Madonia, Greg, and Austin C Smith. 2019. “All-In or checked-out? Disincentives and selection in income share agreements”. *Journal of Economic Behavior & Organization* 161:52–67.

Oei, Shu-Yi, and Diane Ring. 2015. “Human Equity: Regulating the New Income Share Agreements”. *Vand. L. Rev.* 68:681.

Orzach, Ram, and Yair Tauman. 2005. “Strategic dropouts”. *Games and economic behavior* 50 (1): 79–88.

Ostrovsky, Michael, and Michael Schwarz. 2010. "Information disclosure and unraveling in matching markets". *American Economic Journal: Microeconomics* 2 (2): 34–63.

Palacios, Miguel, Tonio DeSorrento, and Andrew P Kelly. 2014. "Investing in value, sharing risk: Financing higher education through income share agreements". *AEI Paper & Studies*.

SLH. 2019. "8 Schools That Offer Income-Share Agreements". Visited on 04/21/2020. https://studentloanhero.com/featured/schools-bachelors-income-share-agreements/.

Spence, Michael. 1973. "Job Market Signaling". *The Quarterly Journal of Economics* 87:355–374.

Swinkels, Jeroen M. 1999. "Education signalling with preemptive offers". *The Review of Economic Studies* 66 (4): 949–970.

Weiss, Andrew. 1995. "Human capital vs. signalling explanations of wages". *Journal of Economic perspectives* 9 (4): 133–154.

Wise, David A. 1975. "Academic achievement and job performance". *The American Economic Review* 65 (3): 350–366.

These appendices contain mathematical details and proofs that are not included in the main body for brevity of presentation.

A. Type Report Independence

In the model, we assume that the mechanism is defined independently of the distribution of reported types. This assumption applies for both the monopoly and duopoly cases. Each of the following sections justifies the assumption.

A.1. Monopoly

Proposition 11 demonstrates why it is without loss of generality to assume that a mechanism is defined independently of the distribution of reported types in the monopoly university.

Proposition 11. *Let $D = (x_H, x_L)$ denote the distribution of types reported by students, such that x_H students claim to be H-types and x_L students claim to be L-types. Then when students are maximizing their private utilities, $D = (N_H, N_L)$.*

Proof. D is determined by two factors, both of which are truthful and universal under the university's general problem:

1. **Which students show up to the university.** By the individual rationality constraints, every student receives nonnegative surplus from being admitted to the university, and faces no show-up cost—thus, showing up always yields nonnegative surplus. Thus, every student shows up to the university and $x_H + x_L = N_H + N_L$.

2. **Which type students report conditional on showing up.** By the incentive-compatibility constraints, every student gains weakly higher surplus from reporting their type truthfully. Thus, every student reports their true type and $x_t = N_t, t \in \{H, L\}$.

Thus, $D = (N_H, N_L)$ is the truthful report of all students in the population. □

Note that N_H, N_L are known to the university—thus, the university knows D in advance of choosing a feasible mechanism. As a result, the university always assumes the distribution of reports is (N_H, N_L) and it is without loss of generality to assume that a student faces a mechanism that depends only on their own announced type.

A.2. Duopoly

Under a duopoly, the type report independence assumption can be framed as the assumption that a student faces a mechanism that depends only on their type. This is because the distribution of types reported to the university depends on two factors:

1. Which students show up to the university. By the individual rationality constraints, every student receives nonnegative surplus conditional on being admitted to each university, so every student applies to both universities.

2. What type students report to the university. By the uniform incentive constraint, in a feasible mechanism, every student maximizes their surplus by reporting their true type: this means there is no scope to pay a lower fee by misreporting type, to either university. Thus, every student reports their type truthfully.

As a consequence, the distribution of types reported to each university is always (N_H, N_L): this allows us to take the distribution of reported types as fixed, and characterize the mechanism only in terms of a student's own type.

B. Monopoly Interior Optimum

We split Lemma 1 into three parts and prove each separately. All use the Lagrange multipliers on the university's problem. The university's problem has the Lagrangian form of

$$\begin{aligned}\mathcal{L} = \quad & p_H N_H c_H + p_L N_L c_L + \\ & \lambda_{ICH}[p_H(w_H - c_H) - p_L(w_H - c_L)] + \\ & \lambda_{ICL}[p_L(W(e_L) - c_L - k(e_L)) - p_H(W(e_L) - c_H - k(e_L))] + \\ & \lambda_{IRH} p_H(w_H - c_H) + \\ & \lambda_{IRL} p_L(W(e_L) - c_L - k(e_L)) + \\ & \lambda_S(1 - p_H N_H - p_L N_L)\end{aligned}$$

The FOCs of this Lagrangian are

$$\frac{\partial \mathcal{L}}{\partial c_H} = p_H(N_H + \lambda_{ICL} - \lambda_{ICH} - \lambda_{IRH})$$

$$\frac{\partial \mathcal{L}}{\partial c_L} = p_L(N_L + \lambda_{ICH} - \lambda_{ICL} - \lambda_{IRL})$$

$$\frac{\partial \mathcal{L}}{\partial p_H} = N_H c_H - \lambda_{ICL}(W(e_L) - k(e_L) - c_L) + (\lambda_{ICH} + \lambda_{IRH})(w_H - c_H) - \lambda_S N_H$$

$$\frac{\partial \mathcal{L}}{\partial p_L} = N_L c_L - \lambda_{ICH}(w_H - c_L) + (\lambda_{ICL} + \lambda_{IRL})(W(e_L) - k(e_L) - c_L) - \lambda_S N_L$$

Lemma 2. *In an interior optimal mechanism, the university allocates all seats.*

Proof. By Kuhn-Tucker,

$$p_L > 0 \implies \frac{\partial \mathcal{L}}{\partial p_L} = 0$$

$$\implies c_L N_L - \lambda_{ICH}(w_H - c_L) + (\lambda_{ICL} + \lambda_{IRL})(W(e_L) - c_L - k(e_L)) - \lambda_S N_L = 0$$

$$\implies N_L = \frac{\lambda_{ICH}(w_H - c_L) + (\lambda_{ICL} + \lambda_{IRL})(W(e_L) - c_L - k(e_L))}{\lambda_S - c_L}$$

The LHS is strictly positive, as $N_L > 0$ by hypothesis. The numerator is weakly positive, as $\lambda_i \geq 0, w_H > w_L \geq c_L$ and $W(e_L) - c_L - k(e_L) \geq 0$ by (IR-L). Thus, the denominator must be strictly positive. This means

$$\lambda_S - c_L > 0 \implies \lambda_S > c_L > 0$$

Since the Lagrange multiplier is strictly positive, the associated constraint $p_L N_L + p_H N_H \leq 1$ must hold with equality. Thus, $p_L N_L + p_H N_H = 1$ and all seats are allocated. □

Lemma 3. *In an interior optimal mechanism, (IC-H) holds with equality.*

Proof. By Kuhn-Tucker,

$$c_H > 0 \implies \frac{\partial \mathcal{L}}{\partial c_H} = 0$$

$$\implies p_H N_H - (\lambda_{ICH} + \lambda_{IRH}) p_H + \lambda_{ICL} p_H = 0$$

$$\implies p_H(N_H - \lambda_{ICH} - \lambda_{IRH} + \lambda_{ICL}) = 0$$

Since $p_H > 0$, this means

$$\lambda_{ICH} + \lambda_{IRH} = N_H + \lambda_{ICL}$$

. We know

$$N_H > 0, \lambda_{ICL} \geq 0$$

$$\implies \lambda_{ICH} + \lambda_{IRH} > 0$$

This means at least one of them is strictly positive. However, assume for contradicition that (IR-H) binds: then by (IC-H),

$$p_H(w_H - c_H) \geq p_L(w_H - c_L)$$

$$\implies p_L(w_H - W(e_L) + k(e_L)) \leq 0$$

$$\implies w_H < W(e_L)$$

However, by hypothesis $W(e) < w_H \ \forall \ e \in \mathbb{R}$, so this is a contradiction: thus, (IR-H) cannot bind. As a result,

$$\lambda_{IRH} = 0$$

$$\implies \lambda_{ICH} > 0$$

and so the associated constraint (IC-H) must bind. □

Lemma 4. *In an interior optimal mechanism, (IR-L) holds with equality and (IC-L) does not.*

Proof. By an identical argument to Lemma 3, using $\frac{\partial \mathcal{L}}{\partial c_L}$ instead of $\frac{\partial \mathcal{L}}{\partial c_H}$, we get that

$$\lambda_{ICL} + \lambda_{IRL} > 0$$

This means at least one of the two constraints must bind. This allows for three cases:

1. **both constraints bind.** (IR-L) binds. Then

$$p_L(W(e_L) - c_L - k(e_L)) = 0$$

$$\implies c_L = W(e_L) - k(e_L)$$

Then (IC-L) binds, so

$$_L(W(e_L) - c_L - e_L^2) = p_H(W(e_L) - c_H - k(e_L)) = 0$$

$$\implies c_H = W(e_L) - k(e_L)$$

Thus, in this case,

$$c_H = c_L = W(e_L) - k(e_L)$$

All students are charged the same price, and all seats are allocated, so this yields the university a profit of

$$\pi = W(e_L) - k(e_L)$$

2. **only (IC-L) binds.** Then

$$c_L < W(e_L) - k(e_L)$$

by non-binding nature of (IR-L), and

$$p_L(W(e_L) - c_L - k(e_L)) = p_H(W(e_L) - c_H - k(e_L))$$

$$\implies W(e_L) - c_H - k(e_L) = \frac{p_L}{p_H}(W(e_L) - c_L - k(e_L))$$

$$\implies W(e_L) - c_H - k(e_L) > 0$$

$$\implies c_H < W(e_L) - k(e_L)$$

This means $c_H, c_L < W(e_L) - k(e_L)$, so

$$\pi < W(e) - k(e_L)$$

Thus, this yields strictly lower profit than case 1.

3. **only (IR-L) binds.** Then

$$c_L = W(e_L) - k(e_L)$$

$$p_L(W(e_L) - c_L - k(e_L)) > p_H(w_L - c_H - k(e_L))$$

$$\implies p_H(W(e_L) - c_H - k(e_L)) < 0$$

$$\implies c_H > W(e_L) - k(e_L)$$

Then the university's profit is

$$\pi = p_H N_H c_H + p_L N_L c_L$$

$$\implies \pi > p_H N_H (W(e_L) - k(e_L)) + p_L N_L (W(e_L) - k(e_L))$$

$$\implies \pi > W(e_L) - k(e_L)$$

Thus, this yields the highest profit of all three cases.

Since the highest profit is obtained from binding only (IR-L), and at least one of the two must bind, it must be optimal to make (IR-L) bind only. □

C. Duopoly Proofs

Proposition 12. *A profile is a pure equilibrium in the tuition duopoly only if it is a sorting profile with* $c = W(e_L) - k(e_L)$*.*

Proof. ($\Longleftarrow$) : Consider the three cases of non-sorting profiles, and we will show that none of them can be equilibria.

1. Both universities charge identical and positive tuition, $c_t^A = c_t^B > 0, \ \forall\, t \in \{H, L\}$ and both give admission to at least one type $t \in \{H, L\}$. Let t be the type admitted by both universities: then deviating from c_t^A to $c_t^A - \epsilon$ gains students of type t from university B. There are $p_t^A p_t^B$ such students, and they were previously split evenly so revenue increases by $\frac{p_t^A p_t^B}{2}(c_t^A - \epsilon)$ from the new students and decreases by $\epsilon p_t^A(1 - \frac{p_t^B}{2})$ from the existing students due to the tuition reduction. The deviation is profitable if

$$\frac{p_t^A p_t^B}{2}(c_t^A - \epsilon) > \epsilon p_t^A(1 - \frac{p_t^B}{2})$$

$$\implies \frac{c_t^A - \epsilon}{\epsilon} > \frac{2 - p_t^B}{p_t^B}$$

$$\implies \frac{c_t^A}{\epsilon} > \frac{2}{p_t^B}$$

$$\implies \epsilon < \frac{c_t^A p_t^B}{2}$$

   This deviation is always possible, so no profile of this type can be an equilibrium.

2. Both universities charge identically zero tuition, $c_t^A = c_t^B = 0, t \in \{H, L\}$. Consider the deviation $c_L^A = c_H^A = \frac{1}{2}(W(e_L) - k(e_L))$. Let t be a type such that $p_t^A > 0$: then at least $p_t^A(1 - p_t^B)$ such students attend A. (Note that if $p_t^B = 1$ then A must be admitting the other type under a candidate mechanism, so the following analysis could apply to that type instead. $N > 1$ so $p_t^B = 1 \ \forall\, t \in \{H, L\}$ is impossible.) This deviation increases revenue from them, and it is strictly lower than their wages so they still show up. This deviation is always possible, so no profile of this type can be an equilibrium.

3. Both universities charge different tuition fees, $c_t^A \neq c_t^B$ for at least one $t \in \{H, L\}$. Note that without loss of generality we can assume $p_t^A > 0$, because if $p_t^A = 0$ then c_t^A is irrelevant and we can consider the other type t'. Then $p_{t'}^A > 0$, and if $c_{t'}^A = c_{t'}^B$ we can apply the analysis from case 1, whereas if $c_{t'}^A \neq c_{t'}^B$ then the following analysis applies. Without loss of generality, let $c_t^A < c_t^B$ and. Then A can deviate to $c' \in (c_t^A, c_t^B)$. Since $c' < c_t^B$, this deviation does not lead H-students to switch from A to B, but since $c' > c_t^A$) it does lead to increased revenue from the existing students of mass p_t^A. This deviation is always possible, so no profile of this type can be an equilibrium.

□

Lemma 5. *If $\alpha^A < 1$, then $p_H^A = p_H^B$ in any feasible mechanism.*

Proof. Any feasible mechanism must satisfy the incentive constraints for truthful reporting. The incentive constraint for high-types is

$$p_H^A(w_H - \alpha^A w_H) \geq p_L^A(w_H - \alpha^A w_H)$$

$$\implies p_H w_H(1 - \alpha^A) \geq p_L w_H(1 - \alpha^A)$$

$$\implies p_H \geq p_L$$

The incentive constraint for low-types is

$$p_L(W(e_L(\alpha^A)) - \alpha^A W(e_L(\alpha^A)) \geq p_H(W(e_L(\alpha^A)) - \alpha^A W(e_L(\alpha^A)))$$

$$\implies p_L W(e_L(\alpha^A))(1 - \alpha^A) \geq p_H W(e_L(\alpha^A))(1 - \alpha^A)$$

$$\implies p_L \geq p_H$$

where $1 - \alpha^A > 0$ so it can be cancelled from both sides. Combining these inequalities yields $p_H^A = p_L^A$. □

Proposition 13. *A profile is a pure equilibrium in the tuition-ISA duopoly only if it is a sorting profile with*

$$p_H^T = \frac{1}{N_H}, p_L^I = \frac{1}{N_L}, c_H^T = w_H, \alpha = 1$$

Proof. Consider the alternative profiles:

1. A sorting profile where T takes low-types for $c_L^T = W(e_L) - k(e_L)$ and I takes high-types for $\alpha = 1$. (Any other α in a sorting equilibrium is infeasible by

Lemma 5) This is not incentive-compatible because high-types prefer to lie and claim to be low-types at university T, paying $W(e_L) - k(e_L)$ instead of w_H, so it cannot be an equilibrium.

2. At least one type t is admitted by both universities, and both charge the same effective fee: $c_t^T = \alpha w_t$. Then T can deviate from c_t^T to $c_t^T - \epsilon$ and gain students of type t from I. There are $p_t^T p_t^I$ such students, and they were previously split evenly so revenue increases by $\frac{p_t^T p_t^I}{2}(c_t^T - \epsilon)$ from the new students and decreases by $\epsilon p_t^T(1 - \frac{p_t^I}{2})$ from the existing students due to the tuition reduction. The deviation is profitable if

$$\frac{p_t^T p_t^I}{2}(c_t^T - \epsilon) > \epsilon p_t^T(1 - \frac{p_t^I}{2})$$

$$\implies \frac{c_t^T - \epsilon}{\epsilon} > \frac{2 - p_t^I}{p_t^I}$$

$$\implies \frac{c_t^T}{\epsilon} > \frac{2}{p_t^I}$$

$$\implies \epsilon < \frac{c_t^T p_t^I}{2}$$

This deviation is always possible, so no profile of this type can be an equilibrium.

3. At least one type t is admitted by both universities, both charge different fees: $c_t^T \neq \alpha w_t$. If $c_t^T < \alpha w_t$, then T can deviate to $c' \in (c_t^T, \alpha w_t)$. Since $c' < c_t^B$, this deviation does not lead H-students to switch from A to B, but since $c' > c_t^A$) it does lead to increased revenue from the existing students of mass p_t^A. This deviation is always possible, so no profile of this type can be an equilibrium. An identical argument follows if $\alpha w_t < c_t^T$.

□